

Qualifying Exam of Galactic Astrophysics 2020

1. (1) For a spiral galaxy, please derive the Tully-Fisher relation with its luminosity L and maximum orbiting velocity of stars v . (10%)
- (2) Please derive the Faber-Jackson relation of an elliptical galaxy with its luminosity L and velocity dispersion σ . (10%)
2. (1) The Oort constants (discovered by Jan Oort) A , B are empirically derived parameters that characterize the local rotational properties of our galaxy, the Milky Way, in the following manner:

$$A = \frac{1}{2} \left(\frac{V_0}{R_0} - \left. \frac{dV}{dr} \right|_{R_0} \right)$$

$$B = -\frac{1}{2} \left(\frac{V_0}{R_0} + \left. \frac{dV}{dr} \right|_{R_0} \right)$$

Please derive A , B from the following figure. (15%)

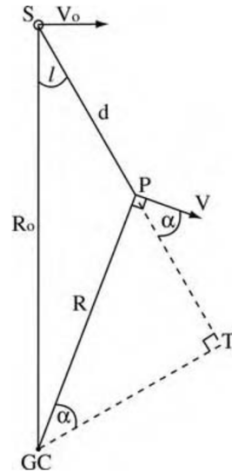


Fig. Galactic rotation: a star or gas cloud at P with longitude l and Galactocentric radius R , at distance d from the Sun, orbits with speed $V(R)$. The line of sight to P is closest to the Galactic center at the *tangent point* T.

- (2) Meanwhile, show that $A + B = -(dV/dr)|_{R_0}$, while $A - B = V_0/R_0$. Show that the IAU values for V_0 and R_0 imply $A - B = 26 \text{ km s}^{-1} \text{ kpc}^{-1}$. Do the measured values of A and B near the Sun correspond to a rising or a falling rotation curve?

What effects might cause us to measure $A + B \neq 0$ near the Sun, even though the Milky Way's rotation speed is roughly constant at that radius? (15%)

p.s. International Astronomical Union (IAU) recommended the values $R_0 = 8.5$ kpc, for the Sun's distance from the Galactic center, and $V_0 = 220 \text{ km s}^{-1}$, for its speed in that circular orbit. To allow workers to compare their measurements, astronomers often compute the distances and speeds of stars by using the IAU values, although current estimates are closer to $R_0 \approx 8$ kpc and $V_0 \approx 200 \text{ km s}^{-1}$.

- (3) Let Ω be the mean angular velocity of disk stars at a distance r from the center of the Galaxy. Please describe in what condition if Ω corresponds to uniform rotation. (15%)
3. Please describe the physical meaning of the *Virial theorem* of a self-gravitating system. (10%) Meanwhile, please derive it for a N-body Collisionless system. (10%)
4. Please derive the Jeans condition for a gaseous, self-gravitating system with finite temperature by linearizing the equations of hydrodynamics. Meanwhile you should explain the criteria of the gravitational stability/instability. (15%)

P.S. The equations you may need to know in this problem:

The hydrodynamical equations of a self-gravitating system,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0, \quad \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = -\nabla \varphi - \frac{1}{\rho} \nabla P$$

$$\nabla^2 \varphi = 4\pi G \rho$$

$$c_s^2 = \frac{\partial P}{\partial \rho}$$