

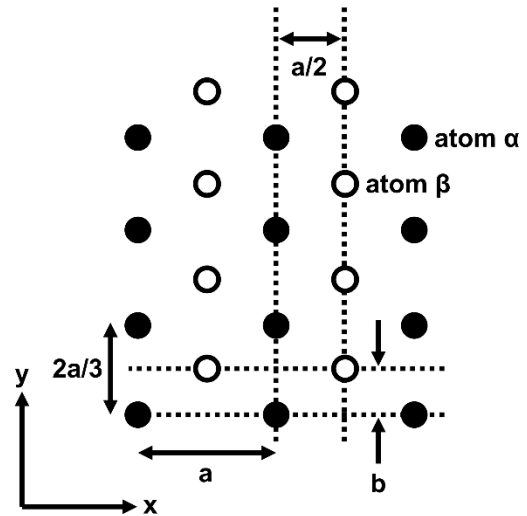
# Qualifying exam: solid state physics

- Note: (1) This is a closed-book exam. Notes, dictionary, calculator, and cell phone are NOT allowed.  
 (2) No one can sit side-by-side with you.  
 (3) Terms and notations follow the textbook of C. Kittel, if not mentioned additionally.  
 (4) If answers have units different from the SI, please describe it explicitly.

## 1. Atomic structure

A two-dimensional atomic structure in the right figure consists of atomic species  $\alpha$  ( $\beta$ ) as indicated by the solid (open) symbols. Use  $\mathbf{e}_x$  and  $\mathbf{e}_y$  as the unit vector along the  $x$  and  $y$  axis, respectively.

- (a) If  $\alpha$  and  $\beta$  are the same element and  $b = a/3$ , derive the primitive lattice vectors  $\mathbf{a}_1$  and  $\mathbf{a}_2$ , draw the primitive unit cell, and write down how many atoms the basis has. (20%)  
 (b) For the case that  $\alpha$  and  $\beta$  are different and  $b = a/6$ , answer the same questions as in the 1(a) above. (20%)



## 2. Reciprocal lattice / diffraction

- (a) Derive the reciprocal lattice vectors  $\mathbf{b}_1$  and  $\mathbf{b}_2$  for the corresponding  $\mathbf{a}_1$  and  $\mathbf{a}_2$  in 1(a). (10%)  
 (b) Answer the same question as in 2(a) but for the atomic structure of 1(b). (10%)  
 (c) Will an incident x-ray with a wavelength of  $2a/5$  be diffracted according to 2(b)? If yes, calculate the angle between the incident and the diffracted x-ray beams. If no, please provide the reason. (10%)

## 3. Electronic structure

- (a) Write down the general Bloch wave function for an electron in a three-dimensional periodic system. Use  $u_{\mathbf{k}}(\mathbf{r})$  for its periodic part with  $\mathbf{r}$  as the position vector and  $\mathbf{k}$  as the Bloch wave vector. (5%)  
 (b) Following 3(a), what is the smallest distance between the neighboring  $\mathbf{k}$  in the reciprocal space if the whole system is a cube with an edge length of  $L$  and the periodic Born-von Kármán boundary condition is applied? (5%)  
 (c) Derive the density of states of a two-dimensional square system with an edge length of  $L$  for spin up free electrons. (5%)  
 (d) In a one-dimensional system with a lattice constant of  $a$  aligned along the  $x$  axis, the electronic structure can be described by the tight-binding model with the  $1s$  orbital of a hydrogen atom. At which location in the reciprocal space does the electronic band have its highest energy? (5%)

## 4. Magnetism / superconductivity

- (a) A ferromagnet possesses a homogeneous magnetization  $\mathbf{M}$  at the low temperature  $T$  approaching 0 K. It has a volume of  $V$  and the Curie temperature  $T_C$ . What is its total magnetic moment at  $T > T_C$ ? (5%)  
 (b) Which of the type I and type II superconductors can allow the penetration of magnetic field into it and the formation of magnetic flux vortices surrounded by the superconducting region? (5%)