

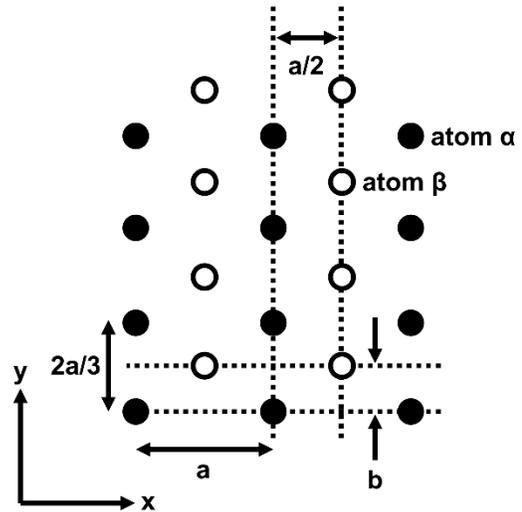
Qualifying exam: solid state physics

- Note: (1) This is a closed-book exam. Notes, dictionary, calculator, and cell phone are NOT allowed.
 (2) No one can sit side-by-side with you.
 (3) Terms and notations follow the textbook of C. Kittel, if not mentioned additionally.
 (4) If answers have units different from the SI, please describe it explicitly.

1. Atomic structure

A two-dimensional atomic structure in the right figure consists of atomic species α (β) as indicated by the solid (open) symbols. Use e_x and e_y as the unit vector along the x and y axis, respectively.

- (a) If α and β are the same element and $b = a/3$, derive the primitive lattice vectors a_1 and a_2 , draw the primitive unit cell, and write down how many atoms the basis has. (20%)
 (b) For the case that α and β are different and $b = a/6$, answer the same questions as in the 1(a) above. (20%)



2. Reciprocal lattice / diffraction

- (a) Derive the reciprocal lattice vectors b_1 and b_2 for the corresponding a_1 and a_2 in 1(a). (10%)
 (b) Answer the same question as in 2(a) but for the atomic structure of 1(b). (10%)
 (c) Will an incident x-ray with a wavelength of $2a/5$ be diffracted according to 2(b)? If yes, calculate the angle between the incident and the diffracted x-ray beams. If no, please provide the reason. (10%)

3. Electronic structure

- (a) Write down the general Bloch wave function for an electron in a three-dimensional periodic system. Use $u_k(r)$ for its periodic part with r as the position vector and k as the Bloch wave vector. (5%)
 (b) Following 3(a), what is the smallest distance between the neighboring k in the reciprocal space if the whole system is a cube with an edge length of L and the periodic Born-von Kármán boundary condition is applied? (5%)
 (c) Derive the density of states of a two-dimensional square system with an edge length of L for spin up free electrons. (5%)
 (d) In a one-dimensional system with a lattice constant of a aligned along the x axis, the electronic structure can be described by the tight-binding model with the $1s$ orbital of a hydrogen atom. At which location in the reciprocal space does the electronic band have its highest energy? (5%)

4. Magnetism / superconductivity

- (a) A ferromagnet possesses a homogeneous magnetization M at the low temperature T approaching 0 K. It has a volume of V and the Curie temperature T_C . What is its total magnetic moment at $T > T_C$? (5%)
 (b) Which of the type I and type II superconductors can allow the penetration of magnetic field into it and the formation of magnetic flux vortices surrounded by the superconducting region? (5%)