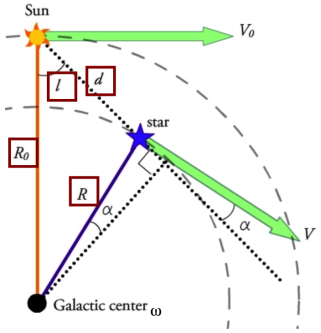
Qualifying Exam of Galactic Astrophysics (2025)

1. Using the attached figure from the problem, please derive the Oort constants A and B. In addition, explain the physical significance of these constants. (30%)



$$A = \frac{1}{2}\left(\frac{V_0}{R_0} - \left.\frac{dV}{dR}\right|_{R_0}\right)$$

$$B = -\frac{1}{2}\left(\frac{V_0}{R_0} + \left.\frac{dV}{dR}\right|_{R_0}\right)$$

2. Please derive the Jeans condition for a gaseous, self-gravitating system with finite temperature by linearizing the equations of hydrodynamics. Meanwhile you should explain the criteria of the gravitational stability/instability. (20%)

P.S. The hydrodynamical equations of a self-gravitating system,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0, \quad \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\nabla\varphi - \frac{1}{\rho}\nabla P$$

$$\nabla^2 \varphi = 4\pi G\rho$$

$$c_s^2 = \frac{\partial P}{\partial \rho}$$

3. Please derive the tensor virial theorem for a system of collision-less particles? (30%)

4. Please explain the physical significance of the Toomre Q factor, emphasizing how it relates to the gravitational stability of a galactic disk. (20%)