Statistical Physics Qualifying Exam (2025)

1. Assuming that the entropy S and the statistical number Ω of a physical system are related through an arbitrary functional form

$$S = f(\Omega),$$

show that the additive character of S and the multiplicative character of Ω *necessarily* require that the function $f(\Omega)$ be of the form $S = k\,\ln\Omega$. [25 points]

2. In the thermodynamic limit (when the extensive properties of the system become infinitely large, while the intensive ones remain constant), the q-potential of the system may be calculated by taking only the largest term in the sum

$$\sum_{N_r=0}^{\infty} z^{N_r} Q_{N_r}(V,T).$$

Verify this statement and interpret the result physically. [25 points]

3. Show that the entropy of an ideal gas in thermal equilibrium is given by the formula

$$S = k\sum_{\epsilon}[\langle n_\epsilon + 1\rangle \ln\langle n_\epsilon + 1\rangle - \langle n_\epsilon\rangle \ln\langle n_\epsilon\rangle]$$

in the case of *bosons* and by the formula

$$S = k\sum_{\epsilon}[-\langle 1 - n_\epsilon\rangle \ln\langle 1 - n_\epsilon\rangle - \langle n_\epsilon\rangle \ln\langle n_\epsilon\rangle]$$

in the case of *fermions*. [25 points]

4. Deduce the virial expansion

$$\frac{PV}{NkT} = \sum_{l=1}^{\infty} a_l \left(\frac{\lambda^3}{v}\right)^{l-1}$$

from

$$\frac{P}{kT} = -\frac{2\pi(2mkT)^{3/2}}{h^3}\int_0^\infty x^{1/2}\ln(1 - ze^{-x})dx = \frac{1}{\lambda^3}g_{5/2}(z)$$

and

$$\frac{N - N_0}{V} = \frac{2\pi(2mkT)^{3/2}}{h^3}\int_0^\infty \frac{x^{1/2}dx}{z^{-1}e^x - 1} = \frac{1}{\lambda^3}g_{3/2}(z),$$

and obtain the following virial coefficients: a_1, a_2, a_3, and a_4. [25 points]