Problem 1:

- Please give the equation of motion for a damped harmonic oscillator in 1-dimension (10 pts).
- Please write a Lagrangian whose Euler-Lagrange equation gives the above equation of motion (10 pts).
- Write down the Hamiltonian for this system (10 points)

Problem 2:
Consider again the one-dimensional harmonic oscillator. The Hamiltonian is given as

$$H = \frac{p^2}{2m} + \frac{kq^2}{2} \tag{1}$$

where $k = m\omega^2$. Let's consider the following change of variable

$$q = \sqrt{\frac{2P}{m\omega}} \sin(Q), \quad p = \sqrt{m\omega P} \cos(Q) \tag{2}$$

- What is the Hamiltonian in the new coordinates ? (5 pts)
- Is the transformation canonical ? Prove it. (15 pts)
- Use the Hamiltonian in the new coordinates and solve for the equation of motion. Use the solution in the new coordinates and solve for (p, q) (10 pts)

Problem 3.
Consider the two-dimensional Kepler problem. The lagrangian is given as

$$L = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{k}{r} \tag{3}$$

- Show that $\ell = mr^2\dot{\theta}$ and $E = \frac{\dot{r}^2}{2m} + \frac{\ell^2}{2mr^2} - \frac{k}{r}$ is a conserved quantity (10 pts)
- Use the Euler-Lagrange equation to derive the function $r(\theta)$, i.e. r is a function of θ (20 pts)

Problem 4.
We define the Hamiltonian as

$$H = p\dot{q} - L(q, \dot{q}) \tag{4}$$

Please explain why the Hamiltonian is a function of p, q only (5 pts). Show that if

$$\{H, G(p, q)\} = 0 \tag{5}$$

where $\{\cdots\}$ is the Poisson bracket, then

$$\delta q \equiv \{G(p, q), q\} \tag{6}$$

is a symmetry of the Lagrangian. (5 pts)